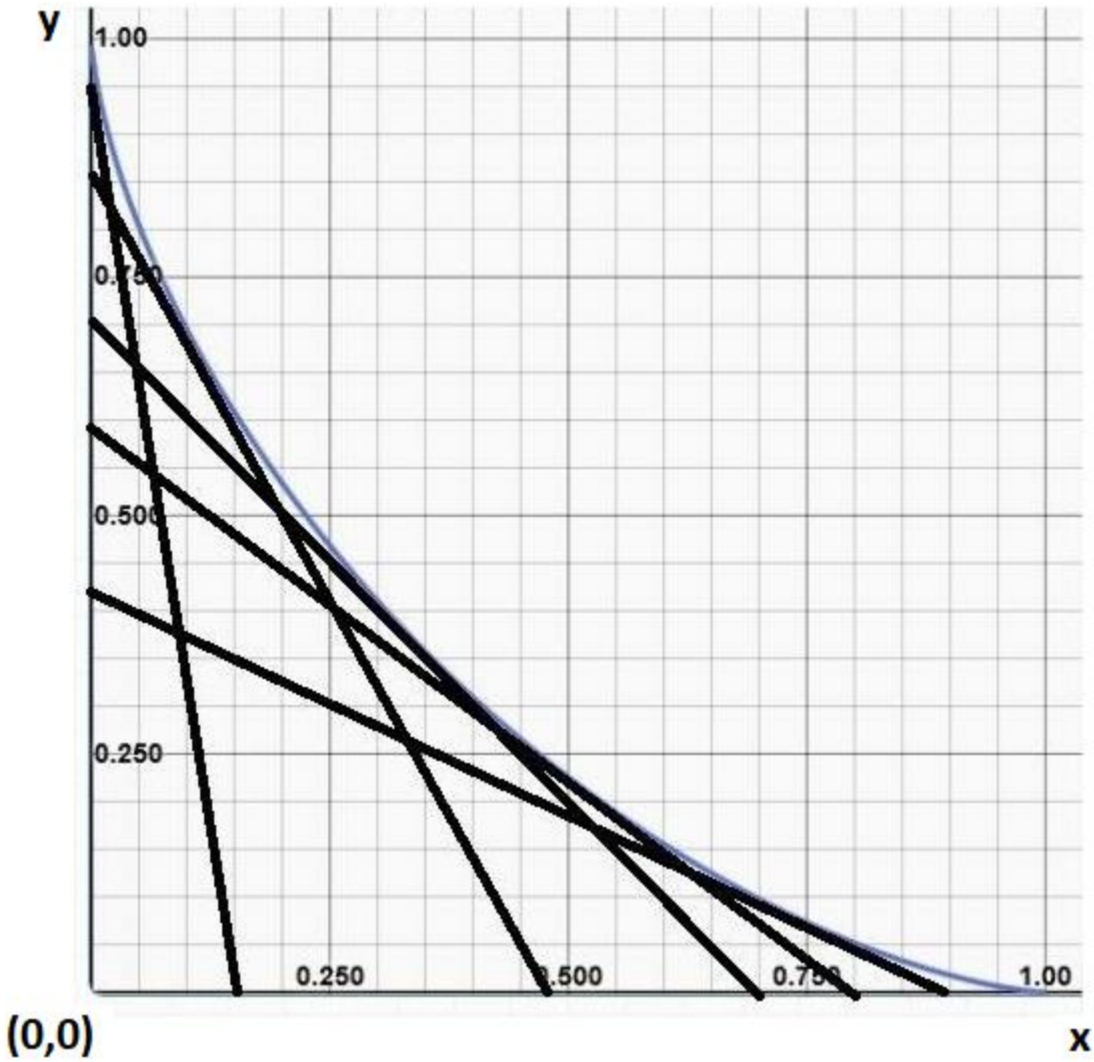
Problem

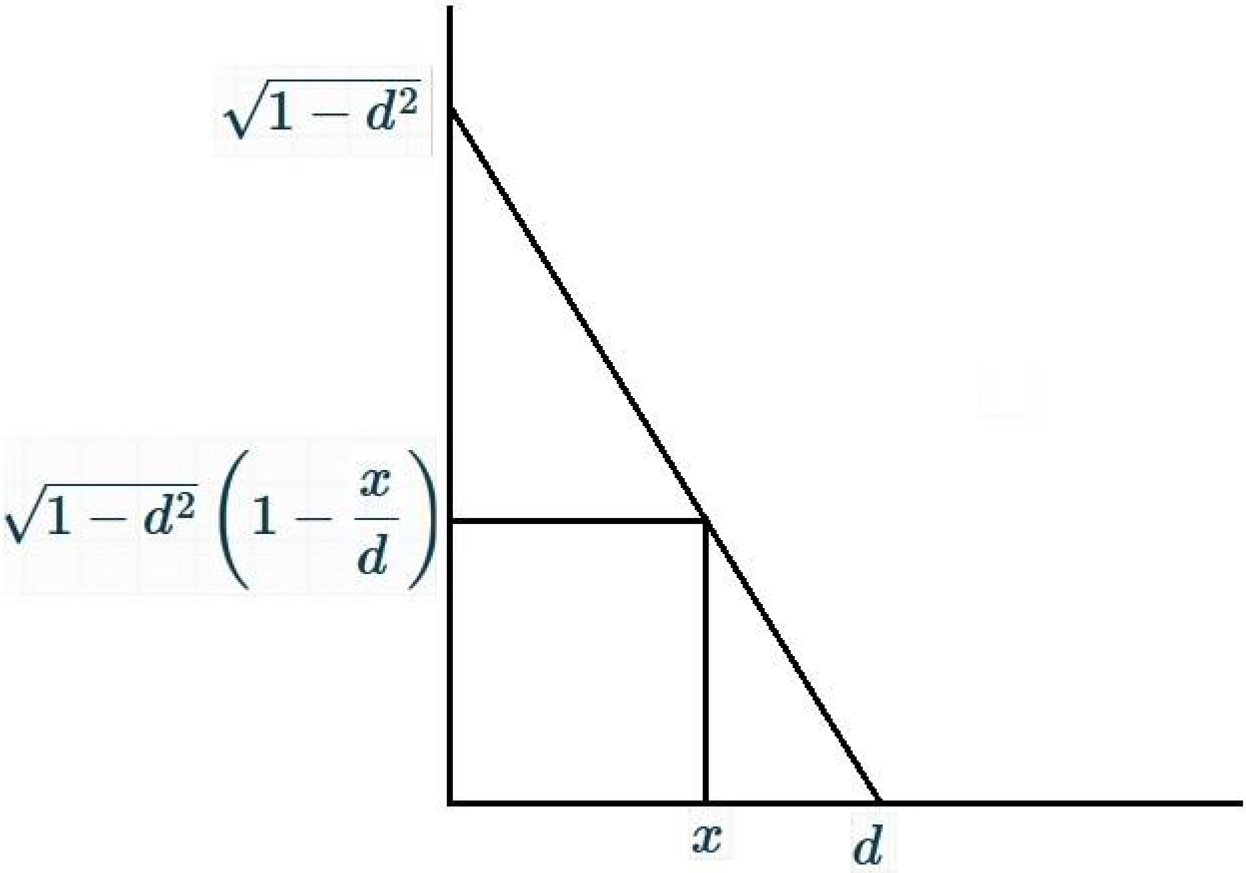
You have a segment with length = 1 and you put it in all possible positions leaning against the axes, like in this image:



A curve emerges, how to express it as y = f(x)?
Stop reading here if you want to solve it yourself.

My solution

Let's calculate some basic values when the segment lowest point is located at (d, 0).
And let choose x between 0 and d:



The highest segment point is located at (0, $\sqrt{1-d^2}$), the ordinate value being calculated using Pythagoras's theorem.

For a fixed value of d, it is easy to calculate y(x), on the segment, using the Basic Proportionality Theorem. The point in the segment is located at (x, $\sqrt{1-d^2}(1-\frac{x}{d})$).

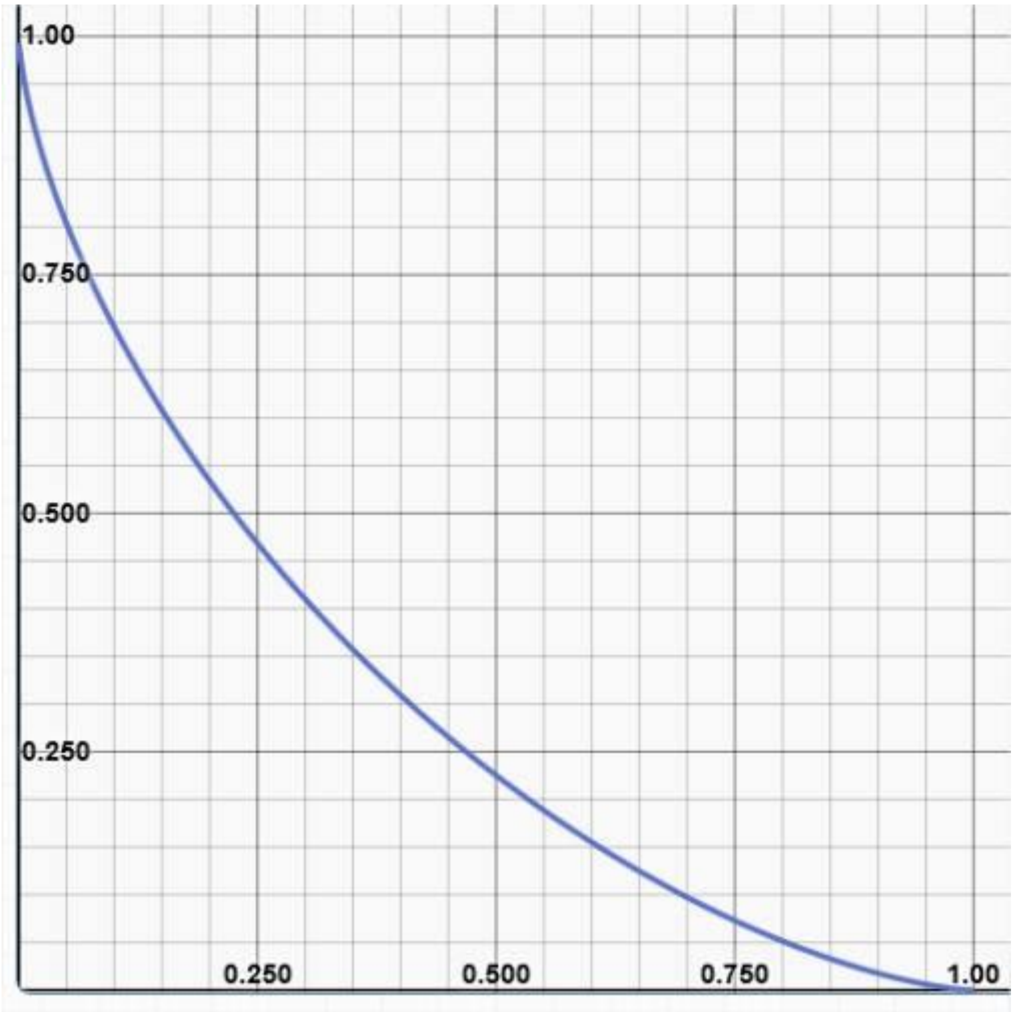
Now, the main idea: we have to find the max y(x,d) for a fixed value of x (with d between x and 1). We do this by setting the derivative $\frac{d(f(x,d))}{dd} = 0$.

I'm not writing all the calculations here, but we finally obtain:

$$-\frac{d^3 - x}{d^2(1-d^2)} = 0$$

The max value is then obtained for $d = x^{\frac{1}{3}} = \sqrt[3]{x}$.

So, replacing $d = x^{\frac{1}{3}}$ in $y = \sqrt{1-d^2}(1-\frac{x}{d})$, we obtain : $y = (1-x^{\frac{2}{3}})^{\frac{3}{2}}$, which is only defined for x in [0, 1] :


1.00
0.750
0.500
0.250
0.250
0.500
0.750
1.00